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1313 Hall of Commerce
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National Oceanic and Atmospheric Administration
National Severe Storms Laboratory

Date: Starnes, Dusan Znic, Mangelore Zachlandad

Considerations for NEXRAD
Doppler Radar Dual Polarization

DRAFT

Znic
## Executive Summary

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Appendix: Simultaneous Dual Polarization Measurements

A.1) ZDR measurement criteria compatible with NEXRAD

A.2) ZDR measurement with simultaneous sampling
A new approach requiring substantial engineering, measurement, and evaluation is required in existing NEXRAD capability necessary to accommodate the dual polarizations. The impact on the baseline system prime mission are examined, possible compromise and the meteorological benefits to NEXRAD are assessed and the interpretation of statistical properties will result from future studies.

It is not anticipated that any significant changes in either physical or mental work necessary to validate the concepts behind the scheme has been established. The data base for evaluation is limited but most of the funnels are becoming well defined and the hardware performance will premature to consider the technique at this time especially since the measurement was not complete with proven meteorological measurement capabilities. However, the work to section at orthogonal polarizations is not an operationally established technique. Differential reflectivity, i.e., the ratio (in dB) of apparent radar cross-section at orthogonal polarizations, is not an operationally established technique in the dual polarization and the associated fundamental measurement, which should be noted that a key word in the assessment is the potential engineering technology necessary to support it.

Recent work in both meteorological measurements using this method and the adjacent features considered but deleted from initial requirements.

Improving certain meteorological measurements by radar and was among the system. Dual polarization has been recognized as a potential method for radar (NEXRAD); several features which would enhance the system capability in formulating technical requirements for the next generation weather radar.
Minimizing the statistical significance of these results.

Summary results are encouraging the database is still too small for definite
identity of the phase state of the precipitation using 2PR data and although pre-
ting the accuracy of precipitation rate estimation. There have been efforts to
have indicated the potential usefulness of these measurements for improving
depolarization ratio (CDR) in the case of circular polarization, the results
depolarization ratio (LR) in the case of linear polarization and circular
polarization method are confined to differential reflectivity (ZDR) and linear
polarization about the scatterers. Though the reported measurements using dual-

hase led to the use of dual polarization in radars to extract additional infor-

study of the polarization dependent scatter properties of hydrometeors
accurately from reflectivity measurement alone.

Volcanic drop diameter, and thermodynamic phase state, cannot be determined
precisely. Certain parameters related to precipitation rate, such as median

great deal of information they do not characterize a meteorological field com-
variance in the radar sample volume. While these spectral moments provide a

the radar scatterers in terms of reflectivity factor, velocity and velocity
the first three spectral moments. These estimates contain information about
Doppler radar with single polarization transmission provides estimates of

2.1. Theoretical Information Contained in the Measurement

2. DUAL POLARIZATION METHOD

This report is intended to serve as a source of information for NEXRAD
into the NEXRAD without serious compromise of the prime mission.

development appears to be the only way dual polarization could be incorporated.
Scattered electric field can be expressed in terms of the scattering matrix. The two orthogonal components of the scattered field, with the backscattered field, hence, it is sufficient to measure such as a raindrop, snowflake, hailstone, etc. A monostatic radar such as NEXRAD measures only the backscattered signal.

Consider the electromagnetic field scatter property of a single hydrometeor.  

2.1 Linear polarization

...derive information content.

...has advantages and disadvantages in terms of practical implementation and the lead to the same results using linear or circular polarization, each method measures the complete scattering matrix, in which case the partial measurement matrix transformation and vice versa. However, it may not be practical to transform linear polarization into circular polarization measurements by two measurements is the same. In this case a set of measurements in linear when a complete scattering matrix is evaluated the information content of the polarization measurement - linear polarization and circular polarization.  

In this section we examine the polarization properties of hydrometeors in order to assess how much additional information we can derive by implementing polarization diversity in a Doppler radar. In practice there are two types of
To understand how the scattering matrix reflects the scattering properties of precipitation rate and cloud conditions, state and drop size distribution, which are essential for accurate detection, and the Thomson matrix. The most useful information is the thermodynamic phase of the scattering matrix. The most useful information is the relative amplitudes of the elements of the scattering matrix (of the resolution volume) and it is possible to extract shape, orientation, phase state and size distribution contoured in the composite of the backscattered power. The information about the scatterer is the expected, and a number of measurements have to be made in order to estimate the scattered field components, which are random. This makes the received signal start.

The radar resolution volume usually consists of a large number of scatterers. The radar resolution volume consists of a large number of scatterers.

The two matrix elements $E_x^y$, $E_y^x$ are proportional to the polarization field $E_z$, while for excitation with a vertically polarized field the two matrix elements $S_{HH}$ and $S_{VV}$ are proportional to the two matrix elements $E_x^y$, $E_y^x$, while for excitation with a vertically polarized field. For an incident field of horizontal polarization, and dielectric constant, the scattered field components $E_x^y$, $E_y^x$ are proportional to $E_z$, and $E_0$, the scattered field components. In general, the elements of the scattering matrix are complex and are functions of the dielectric constant, and dielectric constant, for an incident field of horizontal polarization. In general, the elements of the scattering matrix are complex and are functions of the dielectric constant, and dielectric constant, for an incident field of horizontal polarization. In general, the elements of the scattering matrix are complex and are functions of the dielectric constant, and dielectric constant, for an incident field of horizontal polarization.
established through numerous studies that a free-falling water drop assumes a

shape which can be approximated by an oblate spheroid and at equilibrium.
constant of the drop. Since the eccentricity is a known function of the drop.

Try, particularly, the eccentricity of the ellipsoid as well as the dielectric

Note that the dipole moments $P_d$ and $P_v$ are functions of the drop geometric

$$\text{m} = \frac{\lambda}{n} \text{ Refractive index}$$

$$a/b = \text{axis ratio of the drop}$$

$$\varepsilon = \left[ \frac{1}{\varepsilon_0} \left( 1 - \frac{q}{e} \right) - 1 \right]$$

$$\left[ \frac{\sin \theta}{\varepsilon_0} \right] \left[ \frac{1 - \frac{2}{\varepsilon} - 1}{1 - \frac{2}{\varepsilon}} \right] = 1$$

$$\left[ 1 + \frac{2}{\varepsilon} \left( 1 - \frac{2}{\varepsilon} \right) \right] \left( 1 - \frac{2}{\varepsilon} \right) \frac{\varepsilon_0}{\varepsilon} \frac{\varepsilon}{\varepsilon_0} = P_d$$

Where

$$P_v = \varepsilon$$

$$P_d = P_v$$

$$0 = 0$$

Vertical dipole moments $P_d$ and $P_v$ is given by

as shown in figure 2.1, the backscattered field in terms of the horizontal and

For the ideal drop orientation relative to the incident electric fields

of the sphere, it is a function of the drop size.

(t.e., a drop falling at terminal velocity (or eccentricity) (2.9))
The scattering matrix measured by the radar (Eq. 2.1) represents the number density of drops as a function of prograde and retrograde drop motion axes and incident horizontal field (distribution about a mean φ₀). The distribution N(φ₀) and possibly with a canting angle (φc), the angle between the resolution volume consists of drops of various sizes with a size ratio as a measure of eccentricity of the drop.*

Reflectivities for the horizontally and vertically polarized fields and the strongly dependent on φ, the diagonal terms are proportional to the radar the drop elevation, φ, and orientation, φ, but the off-diagonal terms are more strongly consistent. All four elements of the scattering matrix are functions of the dipole moments Pd and Pw in the major and minor axis direction respectively.

\[
\begin{align*}
\phi_d &+ \phi \cos^2 \phi \left(4d - \frac{3d}{4}\right) \\
\phi_z &+ \phi \cos^2 \phi \left(4d - \frac{3d}{4}\right) \\
\phi_w &+ \phi \cos^2 \phi \left(4d - \frac{3d}{4}\right) \\
\phi_v &+ \phi \cos^2 \phi \left(4d - \frac{3d}{4}\right)
\end{align*}
\]

(2.3)

\[
(2.4)
\]

where

\[
\phi \text{ = elevation angle of the drop} \quad \phi \text{ = angle between n and x, x, y plane}
\]

\[\frac{\phi}{\phi_d} \cos^2 \phi \left(4d - \frac{3d}{4}\right) \]

\[
\phi \text{ = canting angle of the drop (φc)}
\]

\[
\phi \text{ = angle of the drop between n and x, y plane}
\]

\[
\phi \text{ = angle between x, y plane (φ)}
\]

\[
\phi \text{ = angle of the drop from x, y axis to the projection of drop semi minor axis}
\]

\[
\phi \text{ = angle of the drop from x, y axis to the projection of drop semi major axis}
\]

\[
\phi \text{ = canting angle of the drop (φc)}
\]

\[
\phi \text{ = angle of the drop between n and x, y plane}
\]

\[
\phi \text{ = angle between x, y plane (φ)}
\]

can be expressed as (Stauro and Pratt, 1984) for the general orientation shown in Figure 2.1d the scattering matrix provides an estimate of drop diameter at terminal velocity, an estimate of eccentricity of drop diameter for drops falling at terminal velocity.
one of the potential uses of dual polarization measurements may be in the ease of measurement and obtainable accuracies.

the two differential parameters is more suitable for NEXRAD to determine a two parameter DSD. The selection criteria as to which of the two differential parameters, differential reflectivity or differential phase shift, are meaningful is dual polarized radar measurement of one of the different samples measured using a dual polarized radar. The differential phase shift can be extracted from the complex voltage contained in the apparent scattering matrix parameters measured by the system. In the literature, in terms of attenuation and phase shift, the path integrated propagation proper.

the meteorological field to the resolution volume and the scattered field vertically polarized field. Since the transmitted pulse propagates through horizontally polarized electromagnetic field field is different from that for the horizontally that for the plateaus of raindrops the effective propagation constant for the size distribution (DSD) is the differential propagation phase shift. Because another important parameter that can be used to determine accurate drop diagnostic terms is necessary to gain information about this mean canting angle.

value of \( ZDR \) is reduced due to cross correlations, and measurement of the off-diagonal terms is necessary to gain information about the mean canting angle. If \( ZDR \neq 0 \), the size distribution assuming the mean canting angle to be zero. If \( ZDR \neq 0 \), the size distribution provides an unbiased estimate of the two parameters of the drop scatter matrix. The ratio of the diagonal terms, the differential reflectivity, diagnostic could be determined using only the diagonal terms of the case of the mean canting angle \( \theta \) is nearly zero. A two-parameter drop size preferred orientation with their major axes along the horizontal and in most preferred orientations with their major axes along the horizontal and in most
As in linear polarized transmission, the circularly polarized signal can be expressed in terms of a scattering matrix \([C]\) of the result of Doppler spread caused by the relative movement of precipitation particles. The result of Doppler spread from mean frequency is small. The departure from mean frequency is smaller than the scattering signal contains than one frequency but the backscattered signal from precipitation is narrow band Gaussian.


\[
\text{matrix wave}(\text{wave})
\]

- difference between the RH and LHC waves (strictly true only for a monochromatic light)
- the minor axis of the polarization ellipse can be estimated from the phase difference between the RH and LHC waves
- The orientation of the RH wave and the cross coupled component is in a RH wave
- For a RH transmission, the main component of the backscattered field is in a RH wave. The backscattered wave can be resolved into RH and LHC components.
- The two orthogonal circular polarizations are the right hand circular and left hand circular (LHC). If a circularly polarized wave is transverse, the two orthogonal circular polarizations are the right hand circular and left hand circular.

\[2.1.2\text{ Circular polarization}\]

- Information contained in the identity of the hydrometeor type.
- However, an examination of the complete scattering matrix may provide additional information. The results of these types of hydrometeors and identifications may be ambiguous.

- For different types of hydrometeors and identifications may be ambiguous, the results to date show that the range of values of \(2\) and \(2\delta\) often overlaps alone (Brindley and Seltzer, 1977; Seliga and Brindley, 1978; Hall et al., 1980).
- Hydrometeors based on different sets of reflectivity and horizontal reflectivity of these scatters. There have been some studies to identify different types of hydrometeors. The approximate range of values of the scattering matrix elements for each of the approximations needed for the scattering matrix elements.
The angle but is affected by the distribution of canting angles. The angle obtained using only RHCP transmission is independent of mean canting (CDR), but the ratio \( \frac{C_{\text{RL}}}{C_{\text{L}}^2} \) known as the circular depolarization ratio are affected differently by the polarization properties of the scatterers.

But individual matrix terms and derived quantities from these terms depend on the resolution matrix, such as the estimated scatter angle \( \Delta \) of the resolution, volume for circular and linear polarization will also contain the same volume.

Viduqa scattering matrices for circular and linear polarizations contain the backscattering matrix elements for the resolution volume. Because the usual large number of measurements we can estimate the expected values of the matrix to be useful for deriving the information about the scatterers. Using the matrix to be useful for deriving the information about the scatterers, we can estimate the expected values of the matrix, \( \hat{\mu} \), and \( \hat{\nu} \) is the transpose of the matrix, \( \mu^T \).

\[
(s)\begin{bmatrix} E_s^F \\ E_s^H \end{bmatrix} = \begin{bmatrix} I - J & J \\ 0 & I \end{bmatrix} \begin{bmatrix} E_s^F \\ E_s^H \end{bmatrix}
\]

\[
(s)\begin{bmatrix} E_s^F \\ E_s^H \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} E_s^F \\ E_s^H \end{bmatrix}
\]

The matrix can be converted to linear polarized components \( E_s^L \) and \( E_s^R \) by matrix transform. Where subscripts \( L \) and \( R \) represent RHCP and LH components. These components

\[
(s)\begin{bmatrix} E_s^L \\ E_s^R \end{bmatrix} = \begin{bmatrix} C_{\text{RL}} & C_{\text{LR}} \\ C_{\text{RL}} & C_{\text{LR}} \end{bmatrix} \begin{bmatrix} E_s^F \\ E_s^H \end{bmatrix}
\]
II

delivered an isolation greater than 40 dB. With the present hardware, couplers are the limiting component. Orthomode couplers and scalar feeds can deliver an isolation greater than 25 dB. In the proposed radar system designs the switchable CR-20 dB with typical isolation of about 22 dB and matched system isolation of about 20 dB. The couplers required can present a sensitivity delivered a guaranteed isolation of channel isolation of greater than 35 dB is beyond the state of the art. Better than 35 dB.

These off-diagonal terms accurately the channel isolation has to be much better than 35 dB.

The elements (Stap and Pratt, 1984) depending on the rain rate. To measure the channel matrix elements, the off-diagonal elements of the channel matrix are calculated to be 20 to 35 dB below the power level in the main diagonal of the cross coupled power in the off-diagonal elements of the scatter matrix. The scattering coefficients over the entire dynamic range. For example, the magnitude between the orthogonal channels and the phase and gain matching of the two channel hardware aspects that affect the measurement accuracy are isolation missions.

Hardware matrices and the usefulness of these measurements to the NexRAD scatter matrix elements and the usefulness of these measurements of each of the six missions. To assess the accuracies involved in the measurement of each of the six missions, we need to assess the accuracies involved in the measurement of each of the six missions. The accuracy of signals and the propagation effect present in the measurements. Interferometric and circular polarized transmission. However, because of the statistical technology exists for the measurement of the complete scatter matrix in 2.2: Measurable quantities from established techniques missions are made alternately.

affects our white ZDR is nearly unaffected if horizontally and vertically trans-
polarization discrimination in the orthomode coupler and dual receiver. This single receiver scheme eliminates the need for extremely good cal pol separations and only the co-polarized signal is received using a single-receiver transmission is alternatively switched between horizontal and vertical.

It may be noted that $Z_{DR}$ is a function of only the diagonal terms of the scatter matrix, hence there is no need to estimate the cross-coupled component. Where integration is over all drop sizes from zero to the maximum drop size,

$$
\frac{\int_{0}^{\infty} S_{HH}(q)dq}{\int_{0}^{\infty} S_{VV}(q)dq} = \frac{Z}{Z_{DR}}
$$

This function of $Z_{DR}$ is defined as the ratio of horizontal reflectivity to vertical reflectivity and is related to the scatter matrix $S_{[\ldots]}$ and drop size $d$.

$Z_{DR}$ measurement provides a rate estimate of rainfall.

Two parameters, $Z_{DR}$ has been used more successfully to increase the accuracy reported dual-polarization measurements are compared to $Z_{DR}$ and $CDR$. Of these, most of the far, through the technical feasibility of designing a polarization agile radar.

There are no reported measurements of the complete scattering matrix so channel cross-coupling in the radar can be compensated for by computation.

Characterization of the radar system so that the bias error produced by measurement of the off-diagonal components will require accurate...
(2.9) \[
\frac{\int_0^2 S_0^2 + H_0^2}{\int_0^2 S_0^2 - H_0^2} = \frac{\text{CDR}}{\text{dBs}}
\]

The CDRs are related to the scattering matrix elements and drop size distribution by the relation. If all the scatterers are oriented in a common direction, CDR is related to the ratio of co-polar to cross-polar power returns, expressed in dBs. The value of the ratio of co-polar to cross-polar power returns is typically estimated as the expected very good. In practice, it has been found that CDR of less than -5 dB cannot be measured smaller (typically -15 dB), the channel isolation also has to be several orders larger.

(2.2) In CDR measurement the transmission is in either RH or LH polarization.

(2.8) \[ \log \text{ratio estimator} = 10 \log \left( \frac{\| \tilde{E} \|^2}{\| \tilde{H} \|^2} \right) \]

(2.7) \[ \frac{\langle \tilde{E}^2 \rangle}{\langle \tilde{H}^2 \rangle} = \frac{\text{SNR}}{\text{dBs}} \]

The ZDR estimators are defined by Brighet, et al. (1983).

ZDR estimators are usually by either the square law estimator or log ratio estimator. These matching conditions for ZDR are computed from the received complex voltage samples \( \tilde{E} \) and \( \tilde{H} \).
the estimation of precipitation rate depends on the alignment of scatterers in
measurement of ZDR is simpler than CDR measurement. Its effectiveness in

stale.

gate correction is mandatory and for a rate of 80 mm/h, correction is imposed
can be ignored if rainfall rate is less than 20 mm/h. For 40 mm/h, a gate-by-
predictions which imply that propagation effects through 10 km of uniform rain
effect must be accounted for. McGuinness et al. (1984) present data and model
state of the art isolation achievable (~25 dB) is marginal and the propagation
nately measuring co-polar cross-polar components using the same receiver, the
small. While the requirement of receiver matching can be overcome by alter-
which limited its usefulness to situations where the propagation effect is
matching. More important, CDR is severely affected by propagation effects
requirements on the radar system in terms of channel isolation and receiver
well as orientation. However, CDR measurement imposes stringent performance
or the scatterers regardless of their orientation, while ZDR depends on shape as
or more information than ZDR, particularly since CDR depends on the mean shape

\[
\frac{\alpha}{\beta} = \frac{Z_{DR}}{Z_{DR}} \quad (2.10)
\]

transformation
}

elements and DSD. For a monodisperse DSD, ZDR and CDR are related by the
ideally, ZDR and CDR are related to each other via the scatter matrix.

2.2.3 comparison of ZDR and CDR measurements

function of drop shape only.

It can be shown that the CDR is independent of mean scattering angle and is a
channels (Worrall et al., 1973 and 1973b; Ougand and Hossay, 1974; Fang and 
measures extensively in order to assess the depolarization effect on communication 

While differential phase shift has been studied by communication engi-

erent for water-coated hail and water drop of the same size.

differential propagation phase shift characteristics may be significa-
cantly different. Backscatter cross sections similar to water drops of the same size but the 
severe thunderstorms. It is well known that hailstones coated with water have 
full than ZDR. An important use could be for hail discrimination in 
a given hydrometeor type. A combination of ZDR and \( \psi \) may provide more use-

behaviour of these two parameters in relation to ZDR may vary significantly. For 
full in hydrometeor identification if both ZDR and \( \psi \) are computed, since the 
ZDR, \( \psi \) is a parameter of the forward scatter matrix. If \( \psi \) may prove to be use-

can be estimated from the same time series used for ZDR estimation. Unlike 
that the measurement of \( \psi \) does not involve additional microwave hardware. It 
can be used to calculate the two unknown parameters of DSD. It may be noted 
dependent on hydrometeor shape and orientation and \( \psi \) in combination with ZDR 

differential propagation phase shift constant \( \psi \) is also a differential measure 

2.4.4 Differential Propagation Phase Shift Constant, \( \psi \), Measurement 

would be biased by the canting angle of the drops along the propagation path, 

true mean canting angle of the hydrometeors in the resolution volume, but 

can be estimated. Unfortunately this measured canting angle would not be the 
suited (which increases the complexity of the radar system), a canting angle 
If in addition to ZDR one of the cross coupled components is also mea-

affected by propagation.

The important advantage of ZDR measurement is that it is not seriously 

horizontal and vertical directions, which is usually good in rain. However,
The water content of the resolution volume but accurate determination of water
content, Z, of the resolution volume. The reflectivity is related to the total
power from the meteorological targets and estimates the equivalent reflectivity-
doppler radar with single polarized transmission measures the scattered
radial rain rate measurement.

2.3.1 Rainfall Rate Measurements

In the following sections we discuss how the ZDR information can improve these
potential radar observations. Specifically, rain, snow, and supercooled droplets, etc.
distinguish rain, type most notably discrimination

location of the melting layer.

Rainfall rate both at a point and over area.

provide better measurements for such weather features as: rainfall. If this information is insufficient or accurate the radar could

spectral moments, provides estimates of scattering parameters which normally
scatter rain medium. This information, not available in measurements of Z and
provide information about the geometry and thermodynamic phase state of the

For meteorological radars, the polarization properties of the scatters

2.3. Possible Enhancement of NEXRAD Products

studies.

investigated in order to evaluate the usefulness of $\phi$ in meteorological
accuracy to which $\phi$ can be estimated and the statistics of $\phi$ have to be
parameters in one or their early papers, measurements or aspects such as the

(1978) address the possibility of using $\phi$ for the determination of DSD
researchers have been used to determining rain rate, Seliga and Bringi

Chen, 1982), there are no reported measurements of $\phi$ using meteorological
Bringing 1.976 (or CDR and reflectivity at circular polarization magnitudes 8.10f and \( N_0 \) can be determined by ZDR and ZH measurement (Seliga et al., 2.11)). Assuming the form of the DSD to be M-distribution as given in (2.11)

are not practical.

\[ N_0 = \frac{2}{Z_0} \left( \frac{Z_{DR}}{Z_{ZH}} \right) \]

Rautiainen rates and direct measurements to account for these variations in DSD.

city. These variations in DSD result in errors in the empirically determined from the edge of the storm to main short depending on the vertical wind velocity (1984) observed that in a convective storm the parameters vary considerably

dirizzi, widespread showers, thunderstorms, etc. Furthermore, Passaglia[1] two parameters \( N_0 \) and \( a \) vary considerably for different types of rain such as rain rate in mm/hr. Measurements of values of \( a \) & \( b \) in (1986) show that the value of

\[ R = \frac{9}{8} \times 10^3 \text{ mm}^{-3} \text{ m}^{-1} \]

where \( D \) is the drop diameter and (2.11)

\[ N_0 = 8 \times 10^3 \text{ mm}^{-3} \text{ m}^{-1} \]

(Marshall and Palmer proposed that, 1948). Based on their measurements and those of Laws and Parsons most widely used is the two-parameter exponential distribution (Marshall and Palmer, 1948). The most notable among the proposed drop size distributions and probability the rates which differ by as much as a factor of 4 (Dovjak & Zrnic, 1984).

naturally occurring drop size distributions having the same \( Z \) have rational

it has been established that a large variability in the DSD exists, and with discrepancies. Through numerous measurements reported in the literature 1973). Several authors have determined these constants from the DSD measured relationships such as \( Z = aR^b \), where \( a \) and \( b \) are empirical constants (Battan, 1979).

The convective method has been to use an empirical Z-R size distribution. From radar reflectivity requires knowledge of drop content and rain rate, \( R \), 1.976 (or CDR and reflectivity at circular polarization magnitudes 8.10f and \( N_0 \) can be determined by ZDR and ZH measurement (Seliga et al., 2.11)). Assuming the form of the DSD to be M-distribution as given in (2.11)
2.3.2 Location of the Melting Layer

Rather stringent accuracy requirements imply long dwell times for estimation. For ZDR error of 0.1 dB will result in a rainfall rate error of 20%. These in more realistic terms, for Z of 45 dBZ and an error of 0.5 dB then vary from 25 to 27 mm/h which is about 8% of the mean.

Reflectivity of 45 dBZ and a ZDR between 1.9 and 2.1 dB the rain rate would in the ZDR estimate increase at smaller ZDR values. With a fixed rate, examination of the relationship between Z, ZDR, and derived rate the parameter which determines which drop sizes contribute most to the rain measurement ZDR must be estimated to within a fraction of a db because it is estimated by radar. However, to achieve good accuracy in the derived rate thus the additional information in the ZDR could improve the rain rate for the ZDR.

estimation and also reduces the dispersion from 34% using empirical Z-R to 14% a gamma distribution removes virtually all systematic bias in rainfall rate leads to a consistent over-estimation of rainfall rate. They demonstrated that site by site that the assumption of an exponential drop size distribution rain gauges. Furthermore, Cherry et al. (1984) have refined the DSD hypoth-

lics by showing that the assumption of an exponential drop size distribution with ZDR as compared to the empirical Z-R method. Seliga et al. (1981) report data, show that improvement in dispersion of rain rate measurement is possible were compared to the measured rain rates using rain gauge data and disrometer in which the rain rates derived from differential reflectivity measurements consider only the ZDR, ZRH technique. Studies by Seliga et al. (1980, 1981), et al. (1984). Because of its superior immunity to propagation effects we
useful for rain rate measurement and has more potential benefit for NEXRAD.

dual-polarization radar is either ZDR or CD. The ZDR measurement is more

color polarized-radar systems. The most commonly measured parameter using
dual-polarization capability has been implemented both in linear and cir-

3. METHODS FOR DUAL POLARIZATION MEASUREMENTS

large super-cooled raindrops which may be hazardous to aviation.

research groups. Another practical use of Z-DR patterns may be in locating

surface. This new technique is just now being investigated by several

profile of Z, ZDR below it could indicate the potential for hail damage on the

1984) the location of hail signature (Z<55 dBZ and ZDR<0.5 dB)

a dB whereas in hail ZDR is near 0 or negative (bright, cold, and dry).

30, 40, and 50 dBZ, ZDR measurements in rain typically vary between 0.5 and

not been well established yet. Low, medium, and high Z correspond roughly to

between Z and ZDR for various hydrometeors. Quantitative relationships have

Table 1 from Hall et al. (1980) shows qualitative relationships

can be used to infer the type of hydrometeors responsible for the scat-

Measurements of reflectivity-dependent and differential reflectivity

2.3.3 Identification of hydrometeor type

the thermodynamic characteristics of the melting layer and its thickness.

distance between reflectivity peak and ZDR peak may provide information about

have 90% melted (Montgomery et al., 1984). This information together with the

is narrowest and hence delineates very precisely a height where hydrometeors

the peak of CD and 100 m below the peak of ZDR. The ZDR peak

fits as follows: highest is the reflectivity peak, about 100 m below is

the zero degree isotherm. Generally the order of peaks in the vertical pro-

It has been observed that Z, ZDR and CD have relative peak values below
No text or figure on this page.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>ZDR</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZDR Value</td>
<td>4 dB</td>
<td>10 dB</td>
</tr>
<tr>
<td>Z Value</td>
<td>high</td>
<td>low</td>
</tr>
<tr>
<td>Relative Scale</td>
<td>high-density dry hail or other large particles</td>
<td></td>
</tr>
<tr>
<td>Wet hail</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Wet graupel</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Sheet/wet snow</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Dry snow flakes</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Cool drops</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Fog, or super Drizzle, cloud</td>
<td>high</td>
<td></td>
</tr>
<tr>
<td>Rain</td>
<td>high</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1.** Expected characteristics of Z and ZDR at 10-cm wavelength for various hydrometeor types (from Hall et al., 1980).
that affect the drop size distribution and drop fall speeds.

These variations are due to the microphysical and kinematic processes
Seliga et al. (1980) report a range to radar ratio of 0.62 with dispersion of
mean gauge to radar ratio of 1.05 for 14 storms with a dispersion of 30%.

Variation between storms, for example, Birdie and Stramans (1979) report a
empirical Z-R relationship provides a good estimate of the mean but a wide
range measurements. Single polarized radar measurements of rain rate using
significant potential benefit of ZDR to NEXRAD would be to improve rain

3.1.1 ZDR Accuracy Requirement

sensitivity of the computed rain rate to errors in ZDR and ZH or ZV
with the limited number of contiguous ZH, by sample points available and the
information, we need to consider the accuracy with which ZDR can be measured
ZDR measurement scheme can be adapted to the NEXRAD system and provide useful
received NEXRAD radar. For such a scheme, one need to decide whether the existing proven
samples required to accurately estimate ZDR is very large (256 pulses with 10
samples as described in section 3.2.2, and practical experience is that the number of
using the alternate transmission of horizontal and vertical polarized fields
All the measurements of ZDR available in the literature have been made
samples available per beam width is about 48.

Scan rate of 3 RPM, a part of 1 ms, and an antenna 2W or 1°, the number of
and a dwell time of 55 to 42 milliseconds per 1° azimuthal increment. For a
about 15-20 elevation angles. This implies an antenna scan rate of 3 to 4 RPM
inside these five minutes where the volume consists of a full 360° scan at
time less than five minutes where the volume consists of a full 360° scan at
meet the overall mission the system is required to have a volume throughout
Data acquisition time is a prime consideration in the NEXRAD system

3.1 Statistical Considerations
Illustrate the sensitivity of rainfall rate to errors in ZH and ZDR. At a
relationship between ZDR and rainfall rate with NH and ZH as parameters.
The theoretical curves presented in Figure 2.2, which give the

0.1 dB, assuming ZH is accurate to within 0.5 dB.

assumed zero error in ZH and an error of 0.2 dB in ZDR. To estimate rain rate

of 2 dB in ZH. The rms deviation in rain rate is approximately 20% for an

rain rate is a very sensitive function of ZH and ZDR requiring highly

accurate estimates of these parameters. For accurate rainfall estimates.

Figure 4.1. Radar Network.

1 min averaging time is necessary (Masters, 1970). A highly impractical con-
storms. To achieve 5% standard error, a network density of 20 km²/gage with

40% (Milton & Brander, 1979) in the 10-35 km² outflow region of thunder.
error, wind conditions can cause an undercatch estimated to be as much as 20-

means themselves are subject to inaccuracies. In addition to the sampling
the Z-X relationship calibrated by rain gauges. However, rain gauge measure-
dispersion is comparable or smaller than rain rate dispersion obtained from
their results show that the ZDR derived rain rate is the rain
area of 5000 m² were used to compare the ZDR derived rain rate to the rain
standard error in ZDR was 0.28 dB. Data from 317 rain gauges spread over an

taneous radar measurement of rain rate with the ZDR technique. Estimated
calibration of radar measurements using rain gauges with a density of

1979). In one of their complementary radar and rain gauge measurement studies
1000-2000 km²/gauge can reduce the average error to 30% (Milton & Brander,
the mainlobe is at least 15 dB larger than the power through sidelobes. Antennas ZDR will not be affected by sidelobes if the total power received by the edge of storms (Herragh and Carbone, 1994). For typical weather radar beam illuminates weak scatterers near large reflectivity gradients such as at cut and sideline mismatch can cause false ZDR values especially when the main ZDR beam pattern matching in the sidelobe region is in general very difficult.

It is essential that the main beams be matched to avoid any serious bias error in pattern mismatch introduces a bias in ZDR which is difficult to remove. It is affect both ZDR and $\Delta Z$ estimates and produce systematic bias errors. Antennas depolarization and attenuation due to propagation through the rain medium ZDR and thus the calibration error should be considerably better than 1 dB. Calibration does not affect the ZDR. However, it can introduce bias error in used for both horizontally and vertically polarized signals, the receiver is because ZDR is a differential measure and usually the same receiver is transmitted.

Transmissions match between the antenna patterns for horizontally and vertically polarized that propagation phase shift, attenuation in the propagation path, and mis-

errors in $\Delta Z$ and ZDR are receiver calibration, depolarization, and different-

some factors that introduce bias the signal handling process is available. By averaging samples, bias errors can be removed only if a priori knowledge of effects to systematic bias errors. Whereas variance of ZDR and $\Delta Z$ can be reduced in addition to random zero mean errors the $\Delta Z$ and ZDR estimates are sub-

error.

4.0% dB error in ZDR, the estimated rain rate will have a larger than 50% approximately 40% to 50% error in rain rate. With a 1 dB error in $\Delta Z$ and rain rate, whereas at a given ZDR an error of 1 dB in $\Delta Z$ translates to given ZDR an error of 0.2 dB in ZDR results in about 40% error in rainrate.
and $E^\perp$ signal varies between 6 and 45 ms so that an integration time of

\[ \frac{A_d}{H_d} = \frac{\frac{1}{2} |E^\perp| \sum_{i=1}^{3} \frac{W^i}{W} \frac{1}{2} }{\sum_{i=1}^{3} \frac{W^i}{W} } = \frac{\varphi}{\vartheta} \]

(3.1)

et al. (1983) report a median value of the average pulse-to-pulse cross-correlation coefficient of 0.955 and the standard deviation of 0.58 independent samples, between 0.1 to 0.25 dB while the standard deviation of the integration coefficient is 0.68 dB. From 40 parts of the standard deviation of $\varphi$ is a function of the correlation coefficient between $E^\perp$ and $E^\parallel$ signals and the correlation coefficient of the signal, From

(1983) examines here the square law estimator of $\varphi$ is defined as (Bring et al.)

importance in the NEXTRAD application this is the only estimator we will

lowest for the square law estimator. Since minimum dwell time is of prime

that for a given dwell time the standard deviation of the $\varphi$ estimator is

on a "random walk" model to derive the standard deviation. $\varphi$ or $\vartheta$ and concludes

three estimators. Their analysis assume classical Gaussian statistics based

(1983) discusses the statistical properties of the $\varphi$ signal and compares the

"square law," log ratio, or ratio estimator. A recent paper by Bring et al.

$E^\perp$. $\varphi$ is estimated from these samples using one of the three estimators,

The only established $\varphi$ measurement technique uses a polarization switch

3.1.2 Estimation of $\varphi$ by Alternative Sampling of $E^\perp$ and $E^\parallel$
Using (3.4) and (3.2) we can derive an inequality

\[
\frac{Z}{\varnothing (d \mid Z^2)^p} \frac{(I - W)^{-w}}{(I - W)^{3/2}} = \frac{Z}{\varnothing (d \mid Z^2)^p} \frac{H_z}{Z} = \varnothing (d \mid Z^2)^p
\]

A similar analysis for variance of the \( Z \) estimate gives

\[
\frac{\varnothing (d \mid Z^2)^p}{\varnothing (d \mid Z^2)^p + d} = 10 \log_2 \left( \frac{d}{\varnothing (d \mid Z^2)^p} \right)
\]

\( \varnothing (d \mid Z^2)^p \) is in dBs is a normalized (3.2) (d\mid Z^2)^p fractional standard deviation.

relation coefficient between \( \hat{H} \) and \( \hat{L} \) samples for zero lag.

coefficient between \( \hat{H} \) or \( \hat{L} \) samples for multiplex lag and \( \lambda \) is the corre-

lation coefficient between \( \hat{H} \) or \( \hat{L} \) samples, \( \rho(z) \) is the correlation

\[
\frac{\varnothing (d \mid Z^2)^p}{\varnothing (d \mid Z^2)^p + d} = 10 \log_2 \left( \frac{d}{\varnothing (d \mid Z^2)^p} \right)
\]

variance of \( Z \) can be approximated by

variances can be used to predict the behavior of the \( Z \) measurement. The

effect of correlation between contiguous samples in the calculation of spec-

correlation that exists between the samples. Zmic, 1979) examined the

averaging is done with contiguous pairs of samples and one cannot ignore the

that the sample pairs \( \hat{H} \) and \( \hat{L} \) are independent. However, in practice, the

Bringt et al., 1983'), in their analysis of \( Z \)R signal statistics assume

the \( Z \)R estimate (Bringt et al., 1978).

approximately 0.5 seconds is necessary to obtain 0.2 dB standard deviation in
Likely to induce oscillations of drops so that the pair correlation is
for $M = 25$ and $\alpha = 4 \text{ m}^2 \text{s}^{-1}$. However, stronger
influence of $Q^H$ is shown in the Appendix that $\varphi$ is larger with $\varphi_D$ becoming
generally larger than 0.995. Thus the values of $\varphi_D$ given in Figure 3.2
is unsuitable at $\alpha = 6 \text{ m}^2 \text{s}^{-1}$. It is shown in the Appendix that
for $\varphi = 4 \text{ m}^2 \text{s}^{-1}$, for larger spectrum widths the $\varphi_D$ is larger with $\varphi_D$ becoming
value of $\varphi_D$ is 0.12 dB at M = 25 m compatible with NEXRAD specification.
the orthogonal signal correlation (Figure 3.1) when the
limit at $M = 5$ corresponds to $\alpha = 5 \text{ m}^2 \text{s}^{-1}$, and when the $\varphi_D$ decreases with increasing spectrum width at a given $M$, but has a lower
The relations between $M$ and $\varphi_D$ plotted in Figures 3.1 to 3.4 show that
is about 4 \text{ m}^2 \text{s}^{-1}$. The median value for convective storms
to about 10 \text{ m}^2 \text{s}^{-1} in the mesocyclone. The width of weather echoes range from about 1 \text{ m}^2 \text{s}^{-1} for snow and stratiform rain
coefficients are signal properties which are not under our control. Spectral
unambiguous velocity and $T^s$ is the pulse repetition time. These correlation

$$\frac{H}{z} \varphi_D (z) = (z - 4 \text{ m}^2 \text{s}^{-1})$$

The signal correlation function and spectral density constitute a Fourier
of reflectivity below 0.2 dB is sufficient to keep also $\varphi_D$ below 0.1.
The number of samples needed to keep the standard error
from (3.3) (3.3) and (3.5) it follows that for a realistic value

$$\frac{H}{z} \varphi_D (z) = (z - 4 \text{ m}^2 \text{s}^{-1})$$

transformation part. For a Gaussian spectral density $\varphi_D (z)$ can be expressed as

$$\varphi_D (z) = \frac{Z}{(H/z) \varphi_D (z) - Z} < \frac{Z}{(H/z) \varphi_D (z) - Z}$$
Interval for $Z$ would have to be about 3.5 km.

The 0.1 dB criterion for $QR$ to reduce $Z_R$ to 0.5 dB the range averaging corresponds to a standard deviation reduction from 0.17 dB to 0.07 dB at a $g$ of 4 m/s^1 and from 0.24 dB to 0.1 dB at a $g$ of 1 m/s^1 which satisfy reduces the decibel standard error of $QR$ by a factor of 2.5. In dB's, this

a $g$ of 60 km. Assuming a pulse width $T_s = 1.5$ usec, we obtain 6.4 degree antenna. The radar sample volume which height is approximately 1 km at

length (Stimson and Dovetail, 1973). Some averaging of reflectivity is allowed averaging process. The cutoff scale is 2.2 $R_R$, where $R_R$ is the averaging and $Z_R$ scales of interest are greater than the 3 dB cutoff scale of the

In the NEXTRAD some range averaging would perhaps be acceptable provided the $Z$

$$m = g/b + 3/2 \epsilon w$$

depths, $w$, that are averaged by

the number of pulse (number of statistically independent samples) related to the number of pulse approximates a continuous integrator provides a variance reduction factor $M^e$ in both the estimate of reflectivity. A continuous integrator in range

Spatial averaging is a means of reducing the reflectivity and differen-

method must be used.
Mean velocity and width estimates made from alternate transmissio
however, Doppler spectral moment and ZDR are not waveform compatible.
mates will probably be quite different for the two parameters.
waveform compatible although the acquisition times needed for accurate esti-
taneous. Thus, Doppler spectral moment measurement and CDR measurement are
tion scheme since each echo provides the LH and RH polarization powers simult.
If CDR is the parameter of interest it can be obtained from any acquisi-

3.2 Acquisition Waveforms

ZDR are within 0.2 dB and 0.1 dB respectively.

of 180° (see Figure A.18). Under these conditions the bias errors in ZH and
path length of 40 km is needed to produce a 2-way differential phase shift
than 180° even in severe storms. At 3 GHz a 50 mm-1 rain over a propagation
canting angle to be greater than 5° nor the differential phase shift greater
much as -0.5 dB for @ = 180° and @ = 8°. However, we do not expect the mean
Figure 3.5 shows that ZH is underestimated if @ = 0° and the error can be as
given in Figures 3.5 and 3.6. These values are calculated at ZDR = 3 dB.
and ZDR estimates. The bias errors, ZH and ZDR due to propagation are
differentially, in an alternate sampling scheme, resulting in bias errors in ZH
angle, @. The cross coupling affects the measured H and E signals
the two-way differential propagation phase shift, @, and mean canting
field. It is shown in the Appendix that cross coupling is a function of both
causes depolarization and couples power into the orthogonally polarized
horizontally and vertically polarized waves. This difference in propagation
small axes along the vertical. Thus the propagation constant is different for
radarops are orthogonal in shape and tend to orient themselves with the

3.2.3 Bias Error in ZH and ZDR due to Propagation Through Rain
Fixed polarization which results in an increase in the estimate variance of
for spectral moment estimation is reduced to one-half of available from a
and use of multicasting storage as implied in Figure 3,8. The number of pairs
extended to two range intervals by knowing the range distribution of echoes
time in equal number of pairs for $\gamma$ and $\bar{\gamma}$ estimations. Estimates are
The scheme shown in Figure 3,8a splits the samples available in the dwell

\textit{Distribution of the propagation path to this shift.}

the autocorrelation. It is very difficult to account for the cummulative con-
moment estimates would be large because the cosine term in (3.8) attenuates
+90$ degree. Furthermore, when $\theta$ is near 90$ the errors in Doppler spectral
ambiguity in the phase of the autocorrelation whenever $\theta$ is outside a -90$ to
is the Doppler shift, and $\theta$ is the sampling interval. The term $\cos \theta$ causes an
$\theta$ where $\gamma$ is the total differential phase shift between the two components,

\begin{equation}
(3.8)
\phi = \frac{e^{i \theta}}{R(t)} R(t)
\end{equation}

Even worse the mean autocorrelation from such a scheme would be
ratio results in an increase of standard errors in spectral moment estimates.
magnitude of the change ($\gamma$). This decrease in equivalent signal to noise
mean Doppler shift). The signal to noise degradation depends on the relative
amplitude. A decrease in the signal strength (from pulse to pulse) which is reflected in the free-
change in signal strength (from pulse to pulse) which is reflected in the free-
In this simplest form the amplitude modulation can be viewed as a periodic
estimate would be systematically biased.

different polarization. In addition to an increased variance the width
error due to the modulation produced by unequal amplitudes of echoes with
polarization (HAYAHAYA) as shown in Figure 3,7 would have an increased standard
For unambiguous measurement of range, averaging of 1 km. Note that no allowance is made for an interlaced low PRF would be about 1.4 - 0.1 s for velocity and width and 0.1 dB for ZDR with range moment estimation with a 48 sample dwell. The estimate standard deviations providing 36 sample pairs for ZDR estimation and 12 sample pairs for spectral would improve the estimate of ZDR at the expense of the spectral moments By at the expense of other aspects. For example, the scheme shown in Figure 3.4c other schemes can be devised which improve some aspect of the estimation rate estimate over the empirical Z-R technique.

- Range averaging of 1 km would be greater than the 0.1 dB required for improving the correlation activity would be greater than the 0.1 dB required for improving the correlation

- Range averaging of 1 km, the standard deviation of the differential reflectivity would be greater than 1.2 m/s instead of 1 m/s. In addition, even with a data handling, but the mean velocity and width estimate standard deviation of the true moment estimate over two ambiguous range intervals with modification of function with an unambiguous range distribution waveform would provide spec.

- In summary, compared to the present NEXRAD methods this scheme in can:

  - about 2 and resulting standard deviation is increased by about 20%.

  - about 2 and resulting estimate standard deviation is increased by about 20%.

  - the number of samples available for velocity and ZDR measurement is reduced by about a factor of 1/2. The increase in variance of V and W would be greater
averaging in range to 3 km.
Reduction in antenna rotation rate by a factor of two and increased
system capability which may be considered are:
Two alternatives in operational schemes and the resulting impact on the

3.3.1 Modified Operational Scheme
Requirements. This would require a redesign of the RDA.

Imposing the new requirement of ZDR estimation in addition to all existing
compromising some aspect of the existing system capability. Another is to
methods so as to accommodate both types of measurements which will involve
hardware necessary to make the new measurement and modify the operational

There are two general ways to approach this problem, one is to add the
scheme and its ability to address the prescribed mission.

Another requirement for a signal measurement incompatible with the base data measure-
required throughout but with little or no excess time. Incorporation of the
maximum spectral averaging is done and the system provides the data at the
are about as large as tolerable for reliable product generation; close to
variance estimation schemes are used; the standard deviation of the estimates
term is presented specifically at close to its maximum capability, i.e., minimum
time, the radar system characteristics and data spectral resolution. The sys-
ion of the system, base data accuracies are constrained by this acquisition
the maximum time interval which will meet the forecasting and monitoring mis-
ology. The five to six minutes allowed for acquisition of the data is about

The volume acquisition time of the NEXRAD is dictated by the metro-

3.3 Acquisition Time
Increasing the throughput rate of the ZDR measurement. However, the technique

controllably and vertically polarized return power has the potential for
ZDR and spectral moments. Theoretically, the simultaneous measurement of horiz
moment measurement will require a more efficient technique for measuring both

Absorption of the ZDR measurement in addition to the existing spectral

3.3.2 Engineering Redesign

be for rainfall measurement and locating hydrometeor type near ground.
ZDR measurement at only one elevation angle. Probably the only utility would
the overall throughput time. More serious is the question of usefulness of a

accuracy. However, the 1/2 minutes required for the scan adds directly to

function with some range averaging allows measurement of ZDR with the required

the greater number of samples provided by the slow rotation rate in con-

and low elevation angle.

Measurement of ZDR only during one scan at a slow rotation rate (4°-5°)

short, mesocyclone, etc., which have dimensions of this order.
the utility of the ZDR measurement in features like the gust front, hail
increment of 3 km results in a cut-off scale of about 6 km. This may limit
measurement with the accuracy required. However, the throughput time would be

accurately in spectral or moment estimates to be maintained and provide the ZDR

The increased number of samples provided by this would allow the existing

...
any desired polarization sequence can be implemented. Over the switching is executed such as with the lack of ferrous circuits.

pleader for the received signal and the ZDR or CDY calculator. If full control of two different polarizations, unique signal processing consists of a multi-

antenna primary feed and microwave switching arrangement for delivering the

The microwave hardware unique to a dual linear polarization system is the

3.4. Microwave Hardware

about 3 km is acceptable.

averaging of ZDR over about 1 km in range and 2 km over a range interval of

the square moments with the required accuracy and throughput rate if

the system sampling can deliver an estimate of both differential reflectivity and

ions (section 3.4.3) indicate that with careful design the simultaneous

These and other questions are examined in the Appendix. Preliminary calculations

process such as orthogonal signal discrimination and receiver matching.

phase shift as well as the hardware problems associated with dual signals.

the propagation of the two polarizations such as attenuation and differential

measurement, methods must be derived to handle the problems associated with

throughput time for a prescribed accuracy. However, to deliver a suitable

time required to acquire a given number of samples which shortens the data

vertically polarized return powers has the obvious advantage of decreasing the

vertically polarized signals. Simultaneous measurement of the horizontally and

ists of simultaneous transmission and reception of both horizontally and ver-

technique is examined in detail in the Appendix. Basically, the scheme con-

and, if implemented on NEXRAD, would require a redesign of the RDA. This

is unproven and untested and will require substantial engineering development,
Representing stringent engineering design rather than new technology development, antenna criteria present only a small risk (but no small expense) since they characterize the reflector and feed support apparatus. However, the requirements are close match of the primary feed pattern as well as the blockage requirements. To perform well, the dual polarized system will require a close match of the vertical and horizontal antenna secondary patterns. This in turn pared to total radar system cost, there is some engineering risk and time.

Although modifications are straightforward and the cost is modest compared of powers, depending on the algorithm chosen, the ZDR estimates would be from either ratios of power or differences of logarithms. The samples are averaged in both time and range (see Sec. 3.1.2). The antenna is controlled by the transmitted sequence so that echoes behind the receiver is controlled by the common receiver. The multichannel brought above the antenna rotation planes through slip rings, signal process with associated power supplies. The signals to control the switch would be additional waveguide would be installed from the orthomode coupler to the switch. The switch most likely would be situated behind the reflector along an inserted behind the feed to accept either vertical or horizontal fields. An so that either field can be radiated. An orthomode mode coupler would be needed to be added on a NEXRAD. Feed horn must be circular (or a scalar type) and a simplified block diagram (Figure 3.9) shows the components that would Measurement of ZDR 3.4.1
estimates would be smaller, and unlike the ZDR technique, CDR measurements do
obtain energy from each returned echo. Consequently, the standard error of CDR
measurement is not from sequential signals but sample parts of powers are
measurement is about the same as that of differential reflectivity. Note that

The incremental cost of implementing the circular depolarization ratio

the antenna coupler to achieve good isolation with acceptable cross comp-

than linear dual polarization. However, it is possible by careful design of
calculation, CDR estimation imposes a more stringent antenna design criteria
receiver is needed for the orthogonal component and a signal processor for CDR
elliptical polarization into its LH and RH circular components. An additional
sis of antenna feed horn and a microwave circulator to resolve the returned

Referring to Figure 3.10, the hardware needed for CDR measurements can-

3.4.2 Measurement of CDR

Full matrix measurements (diagonal and off diagonal elements) for
ZDR measurements (diagonal elements of the scattering matrix) but marginal for
factoring techniques [ESI, 1983]. An isolation of 20 dB is adequate for the
It is not likely that this will improve with the existing materials and manu-

a guaranteed isolation of about 20 dB with typical isolation of about 22 dB.
meaning risk, is the orthogonal isolation. The present technology can deliver-
ic of the ferrite switch, which is more of a system limitation than an engi-
neering development cost would probably be about $500K. Another characteristics
increase over the lower power units presently in service, the one-time engi-

the production costs of the higher power units would probably be a small
for the high peak and average power (1 MW, 2.5 kW) required of NETRAD. While
switchable ferrite circulator. Most circulators in use to date are not rated

important. There is a significant risk in the design and development of the
The return signal is also switched between two receivers at one half the switching rate. Switching receivers provides inherent compensation of small receiver mismatch errors and relaxes the matching required to a reasonable value for an acceptable bias. Switching at half the transmitter rate prevents the receiver mismatch from contaminating the spectral moment estimation.

The polarization plane at 90° is equivalent to switching a which results in a zero mean odd function of the canting angle \( H_{DR}(\theta) = -\frac{1}{2} H_{DR}(\theta) \) switching canting and differential phase shift. Since these errors are approximately to be taken of the symmetry of the bias error in \( H \) and \( Z_R \) due to drop switching the signal polarization between \( +45° \) and \(-45° \) allows advantage.

Turtles. The receiver outputs are applied to the signal processor by the orthomode coupler and applied to the receiver switch via the T/R circuit. The received signal is decomposed into its horizontal and vertical components.

The transmitted signal is divided in half and applied to the horizontal and vertical components. Both channels and a switchable 0 or 180° phase shifter are switched by either excitation of the antenna feed is oriented conventionally and horizontally and vertically.

The block diagram of the simultaneous measurement system is given in Figure 3.11. The simultaneous measurement of \( Z_R \) is not affected by Doppler spectral moments, however, the susceptibility of this

operational implementation.
Estimation of differential reflectivity (or differential return power in
3.5 Signal Processing Hardware

ZH and ZHR estimates.

Spectral moment estimation and by increasing range averaging interval for the
acquisition rate for a spectral accuracy by using both polarizations for
the simultaneous system would have some capability of increasing the data
miter power and all other parameters equal.

Signal detection capability is dB below an alternate system for a given trans-

Reference

NXRAD modified for alternate sampling, this scheme has the following dis-

NEXRAD model for efficient and environmentally controlled. Compared to the existing
frequency specific and environment range. This component would probably be
operational frequency and temperature range. This component would probably be

The measured data, the shutter is high risk for an accuracy over the NXRAD
and antenna feed. The shutter is high risk for an accuracy in the phase shutter
SNR > 10 dB, at lower SNR the NNR required performance may not be achievable.

The development and imaging risk of this system is in the phase shutter
parts containing a common pulse will result in negligible degradation at
signals of both. Moment estimation from spaced pairs rather than continuous
spectral moment estimation can be done on either horizontal or vertical

Method, by range averaging.

The 0.1 dB needed to improve the spatial rate estimate over the empirical Z-R
0.72 dB; about one half that for alternate sampling. This could be reduced to

(50 samples), the standard deviation of 0.72 for simultaneous sampling is
signal correlation is 0.995, spectrum width is 4 MHz and dwell time is 50 ms.

scheme is given in Figure 3.12. It is seen that when square of the orthogonal

An example of the theoretical 2-2 estimate accuracy achievable with this
It is seen that the hardware requirement and computational complexity is significantly greater than for the intensity integrator and less than for the velocity
sors in the system (Strum's, 1980)

- Characteristics are tabulated (Table 2) in the same fashion as other signal processes

with the estimate being the ratio of average. The processor hardware characteristic

hence are of equal cost. Averaging is done separately for each polarization

these operations require about the same amount of integrated circuits and

\[ E \hat{u} = \text{antilog} (10 \log p U) \]

\( \log p, \) (output of the log receiver) by

samples. I a (output of linear receiver), or the logarithm of echo power,

In the case of NEXRAD, \( E \hat{u} \) can be derived from either the complex video

\[ \frac{A_{d}}{H_{d}} \text{log} 10 = \left( \frac{u}{w} \right) \text{log} 10 \text{DRK} = \text{log} 10 \text{DRK} \]

\[ \text{cation of equation (3.1) with addition of the logarithmic operation, i.e.,} \]

The proposed algorithm for \( \text{DRK} \) calculation is a straightforward implement-

prime consideration in the NEXRAD system.

\[ \text{Acquisition time is of course a} \]

square law estimator (Brungel et al., 1983). Acquisition time is the time the

algorithm is the minimum standard deviation for a given dwell time is the

six algorithms for \( \text{DRK} \). However, as noted in Section 3.1, the most efficient

ensemble averaging (average of ratios and ratio of averages) can only derive

response types (square law, linear, or logarithmic) and two techniques of

practices can be done in a variety of ways. With the three basic receiver
stantial changes in either operational methods in conjunction with basic
malleted fundamentally to measurement of the spectral moments will require sub-
implementation of the orthogonal polarization measurement on a radar com-
not be necessary to realize the prime benefits of the technique.
eters, is marginal with existing hardware but full matrix measurement would
measurement, i.e., estimation of the off diagonal as well as the diagonal element-
measurement without new engineering development. Full scattering matrix mea-
art engineering can provide the microwave hardware necessary for the basic
fall rate estimates and discrimination of the scatterer phase. The state of the
Differential reflectivity has some potential for improving NEXRAD rain-
network radar product generation.
meteorologically the measurement would probably find little application in the
80 mm/hr. Since these are not the regions of primary interest to the radar
lessthan that corresponding to propagation through 10 km at rainfall rate of
regions of storm systems with low rainfall rates where the composite effect is
NEXRAD. Propagation effects will limit the usefulness of this measurement to
CDR measurements are of limited utility and questionable benefit for the

distinguishing the type of scatterer (i.e., ice or liquid water).
an empirical reflectivity rainrate relation and it could also aid in
cular this information could improve therainfall rate estimate over that of
potential for improving the capability of the NEXRAD network radar. In parti-
tation, provided by measurements at orthogonal polarizations ZDR, has the
In general, the information about the radar cross section of the scat-

3.6 Summary

Estimation of CDR involves essentially the same hardware consideration.
city and which calculation.
longer acquisition time for the composite measurement. For example, at ZH, the
inherently longer dwell time for the ZDR estimator causes a significant
loss in signal-to-noise ratio. Since the sequence cannot be optimized for either, the
results in a longer total dwell time for a given statistical accuracy in signal
sequence is necessary to accommodate both type measurements. This in

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<td>1024</td>
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Computations
Output word length
Throughput rate
Total memory
Storage word length
Number of range locations
Input word length
Algorithm

Differential Reliability Calculator
Table 2
ever, this technique is unproven and would require extensive engineering.

The estimates with the accuracy needed at the required throughput rate. How-

conjunction with a 3 km range averaging of ZHR and ZDR could in theory provide

range resolution of ZDR and spectral moments. The simultaneous technique in

which with a dwell time of about twice that in the present NMR for equal

horizontal and vertically polarized echoes could provide the required sta-

Theoretically, the method of simultaneous measurement of powers from

ZHR of about 1/2 minutes adds directly to the volumetric throughput time.

about one fourth that of the operational rate. However, the scan time for

one or two is sufficient. These could be acquired at a slow rotation rate

If the ZDR measurement at only a limited number of elevations (i.e., power

of about 10 minutes and a ZDR measurement on scales greater than 6 km.

two and range averaging of ZHR over 3 km would provide a volumetric throughput

used to achieve the required accuracies. Reducing antenna rotation rate by

A combination of increased dwell time and range averaging could also be

minutes to about twenty minutes.

four. This would increase the volumetric throughput time from about five

rate would have to be decreased (increase in dwell time) by about a factor of

(with equal range resolution of spectral moments and ZHR) the antenna rotation

Thus, if a straightforward implementation of ZHR on the NEXRAD is made

of the total acquisition time is almost the sum of the two

important waveform types. Since these two types of measurements have

ZHR accuracy of 0.1 dB is about three times that required for spectral moments

the NEXRAD parameters the dwell time with alternate polarization required for

accurate to 0.1 dB to provide a reliable estimate less than 20% for

accurate to within 0.5 dB with a mean of 4.5 dB, the ZDR estimate needs to be
teems modification necessary for the ZDR measurement.

Present detection capability in addition to the receiver and antenna subsys-

require a major redesign of the present NEKRAO transmitter to maintain the
development and testing before acceptance. If accepted, the technique would
with alternate sampling scheme.

Fig. 3.4 Bias error in differential reflectivity, $\Delta R$, due to propagation with alternate sampling scheme.

For $|q_A^V(0)|^2 = 0.98$.

Fig. 3.5 Bias error in horizontal reflectivity, $\Delta H$, due to propagation with alternate sampling scheme.

\[
\text{standard deviation of } Z_D, Z_P \text{ versus numbers of sample pairs, } M.
\]

For $|q_A^V(0)|^2 = 0.99$.

\[
\text{standard deviation of } Z_D, Z_P \text{ versus numbers of sample pairs, } M.
\]

For $|q_A^V(0)|^2 = 0.995$.

\[
\text{standard deviation of } Z_D, Z_P \text{ versus numbers of sample pairs, } M.
\]

For $|q_A^V(0)|^2 = 1.0$.

Fig. 3.1 Relation between differential reflectivity, $\Delta R$, and rainfall rate.

The geometry of radar Doppler scatter.
random and alternate sampling.

Fig. 3.12 Standard deviations of differential reflectivity, $\rho_D$, with simultaneous sampling of $E_H$ and $E_V$.

Fig. 3.11 Block diagram of the dual-polarized radar for CDR measurement.

By alternate sampling of $E_H$ and $E_V$ signals.

Fig. 3.10 Block diagram of the CDR polarized radar for CDR measurement.

Fig. 3.9 Possible transmission sequence for NEXTRAD.

Fig. 3.8 Alternate transmission sequence. Correlations of interest are indi-
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This scheme provides vertically and horizontally polarized samples for every simultaneous sampling of $E_h$ and $E_v$ signals as suggested in section (3.3).

with ZDR measurements. One of the schemes that may have such a capability is of retaining the capability to estimate the spectral moments simultaneously with ZDR estimation. If it is not only necessary to derive means of reducing QDR but also estimations, it is shown to be not very practical.

In order to arrive at a measurement scheme compatible with NEXRAD spectral estimates of ZDR estimation.

other sampling sequence (e.g., see Figure 3.8) further increases the variance in terms of simultaneous measurement of spectral moments (see Sec. 3.2). Any Alternative sampling scheme (Figure 3.9) is shown to be not very practical.

pairs are available per ZDR estimation. If alternative sampling schemes are used only 25 samples of IQRs are used, which is less than 50 samples per estimation. If azimuthal averaging is to be less than 50% of the beam for NEXRAD. A scan rate of 1.5°-1 precises, more than about 50

specification for NEXRAD, a scan rate of 1.5°-1 standard deviation of ZDR, the averaging time of the number of samples required is large and cannot be obtained with the scan rate obtained less than 0.1 dB standard deviation of ZDR. The averaging time of the sequence 3, we can conclude that ZDR has to be estimated within 0.1 dB accuracy. From the statistical considerations of ZDR signal as discussed in Appendix A.1, ZDR Measurement Criteria Compatible With NEXRAD Specifications

Simultaneous Dual Polarization Measurements
Bias errors and their computation is considered later in this appendix.

We consider several factors that affect the correlation matrix and its eigenvalues.

In order to assess the range of values that can be expected for the correlation matrices, we need to understand the range of values that are present in the matrices. In order to determine the range of values that are present in the matrices, we need to understand the range of values that are present in the matrices.

There are several problems associated with simultaneous sampling. Important to less than 0.1 dB by spatial averaging over 6 to 9 range samples.

This standard error can be reduced by using a larger number of samples. However, for narrow spectrum widths (\(\nu = 1\) in ms-1) DBR is more than 0.2 dB and a larger number of samples is required.

If the partial correlation better than 0.995 can be expected in practice, spectrum widths between 2 and 4 ms-1 for M = 50. It is shown in Figure 6.1 that the partial correlation better than 0.995, DBR spans 0.2 to 0.1 dB for a parameter. A.3 for \(\nu = 0.0\), A.4 for \(\nu = 0.5\), A.5 for \(\nu = 0.995\), and A.6 for 0.98 respectively, with a spectrum width as seen in Figure 6.1. The same notations are used as in Section 4, plot of DBR expressed in db.

\[
\text{VAR}(\nu) = Z_{DBR}^2 (1 - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2}) \nu = \text{VAR}(H) \text{ but has an additional multiplicative factor. VAR}(\nu) \text{ can be approximated as:}
\]

An analysis of the variance of DBR estimates using simultaneous sampling scheme which provides only half the numbers of samples.

Further, it also gives us as many samples pairs as the number of transmitters.

Pulse transmission, thus retaining the spectral moment estimation capability.
These limits are approximated from the observed maximum and minimum values of the form of variance of the function of evantolometric diameter $a/b$ (Figure A.4). The standard deviation and maximum and minimum truncation values of $a/b$ are.

The standard deviation of $a/b$ about the mean is taken to be $1/3$ maximum amplitude which is also the truncation point of the distribution (Figure A.5). The shapes about the oscillation mean $a/b$ are given by Beard et al. (1983).

The drop shape and oscillation we assume a truncated Gaussian distribution due to the decay rate. For the purpose of computing the decorrelation due to the curve $b$. The oscillation amplitudes are dependent on collision energy and shown that this mean is slightly higher than equilibrium $a/b$ (see Figure 4) have averaged mean $a/b$ rather than the equilibrium $a/b$. Beard et al. (1983) have assumed to have random phases; hence the measured ZDR is proportional to the time oscillations are generally induced by drop collision and break up and can be between oblate and prolate spheroidal shapes about a mean $a/b$ ratio. These will vertically polarized fields and the vector sum of vertically.

This equilibrium model (solid curve a in the Figure) because the distribution shown in Figure A.4 shows the variation of $a/b$ ratio with equilibrium diameter for equilibrium axis ratio $a/b$, is given by Green's equation (Green, 1975).

For a given equilibrium diameter $a/b$ of the drop, the oblate spheroidal shapes and the oblateness of $(q/a)$-axis ratio is a function of the resolution volume consists of a large number of scatterers of different shapes and sizes depending on their thermodynamic phase state. We can-

A.1.1. Non-Spherical Drop Shape and Drop Oscillations
of the drops observed in rain with bigger drops in higher rain rates. We also
drop size distribution with Δθ = 5 mm. Five to 8 mm is the maximum diameter
decorrelation due to canting angle distribution; we consider a monodisperse
independent of θ0. In order to get an idea of the upper limit of the
EV signal's the correlation coefficient $|\rho(\theta)|^2$ is a function of $\theta$ but is
bias error in the measured ZDR but does not contribute to decorrelation of $E_H$
Gaussian with mean $\theta_0$ and standard deviation $\sigma_\theta$. The mean $\theta_0$ introduces a
independent of drop size. The canting angle distribution is assumed to be

The important assumptions he made is that the canting angle is statistically
The distribution model with randomly cantal drops for propagation studies. One of
incidence in the resolution volume. Omacht (1977) has constructed a random
However, in nature, drops can have other orientations due to shear and
was assumed that all the drops are oriented along vertical-horizonal direc-
In the analyses of decorrelation due to drop shape and oscillations, it

A.1.2 Canting Angle Distribution

oscillations.
Rates less than 200 mm/h. Correlation is > 0.995 when the drops are not
oscillating. Drops it can be noted that $\rho(Y,0)|^2$ is better than 0.99 for rain
Due to drop shape and oscillation are presented in Figure A.6. With
The computed theoretical values for the correlation coefficient $\rho(Y,0)|^2$ of
0°C and the corresponding refractive index value

0°C with $N_0 = 8000$. The raindrops are assumed to be at a temperature
distribution $N(D)$ is assumed to be exponential Marshall-Palmer type ($N(D)$)
and $\rho(Y,0)|^2$. Curve c), as reported in the literature. Drop size
0.999. For simultaneous sampling the scanning would not affect the pair 180°-1.6 MW and 1 ms PRF, this correlation coefficient is better than to scan the one-way (-3dB) beam width of the antenna. For 

\[ R(1) = e^{-2\pi n^2} \]  

(4.3)

Relation for computing the correlation coefficient is given by (2.24) and de-correlation time, and hence \( R(1) \) for alternate sampling. An approximate gradient in the ZDR will be averaged to produce a mean value. Even when there is no gradient and the rain field is uniform, scanning reduces the signal gradient in the ZDR will be averaged to produce a mean value. Even when there is no gradient, the rain field is uniform, scanning reduces the signal gradient in the ZDR will be averaged to produce a mean value. Even when there is no gradient, the rain field is uniform, scanning reduces the signal gradient in the ZDR will be averaged to produce a mean value. Even when there is no gradient, the rain field is uniform, scanning reduces the signal.

\[ (4.1) \]

4.1.3. Decorrelation due to antenna scanning.

\[ \text{eccentricity of the drop, reducing the effect of canting on } |H(0)|_2^2 \]

better than this number because the smaller the drop is, the less is the deviation of \( \theta = 180° \) for the rain with exponential DSD. The correlation is 0.95 required for compatibility with NEXRAD even for very large standard DSD is given in Figure A.3. This figure shows that \( |H(0)|_2^2 \) is better than in the computation. The correlation coefficient \( |H(0)|_2^2 \) for mono-disperse an equilibrium shape with \( a/b = 0.65 \) corresponding to a 5 mm drop. It is used

\[ \left( \frac{\theta}{\theta_0} \right) \]

(4.2)

\[ \left( \frac{\theta}{\theta_0} \right) \]

Standard deviation of \( \theta \). The canting angle distribution is given as

Assume Gaussian distribution for the canting angles with zero mean and
incorporated to compensate for the propagation effect which is explained later.

Switching between 45° and -45° with respect to horizontal. This switching is
designed to change the vertically polarized signal so that the polarization plane can be
between two phase settings of 0° and 180°, is introduced in the transmission
separate phase and gain matched receivers. A phase shifter, switchable
controls change the horizontally and vertically polarized signals into two
channels when the horizontal and vertical polarization plane by exactly 45°. A power divider channels equal power into
polarization modes are excited in phase in the antenna feed to rotate the
linear polarization is used. The horizontally and vertically polar-
in in Figure A.7. To sample $E_{\parallel}$ and $E_{\perp}$ simultaneously, pulse transmission at 45°
a schematic of the proposed simultaneous $E_{\parallel}$ and $E_{\perp}$ sampling system is given

A.2. ZDR Measurement with Simultaneous Sampling

Advantages of the scheme.

Consider the possible accuracy of ZDR measurement, and advantages and dis-
advantages of the scheme. We propose a scheme of measurement in the next section and
Comparisons between with the ZDR estimation compatible with the measured 3.8%.
All these considerations establish that the only possible way of making

or $|p(t)|^2$ for one pulse lag with spectrum which is given in Figure A.6.

A.4. Doppler Velocity Spread

gaussian spectrum the correlation coefficient is given by (3.8). The correlation
with time, but does not affect $|A(t)|^2$ because of zero time lag. For a
Doppler velocity spread is mainly responsible for signal de-correlation

A.4.1.4. Decoherence due to Doppler Velocity Spread

Alternate sampling, and hence it can be neglected.

Correlation. The decoherence due to antenna scanning is very small even for

field.

of error can be evaluated using simplified models such as a uniform rain
canting angle and phase state along the propagation path. However, the order
effect exactly for a complex precipitation medium of varying reflectivites,
measured ZDR and ZH. It is extremely difficult to evaluate the propagation
and horizontally polarized propagating fields, leading to this error in the
and horizontally polarized propagating fields, leading to this error in the
hydroxymers in the propagation path cause a cross coupling between vertically
Together the differential propagation properties and the canting angle of the
phase shift for the horizontally and vertically polarized waves differ.
orientation of the scatterers in the propagation medium, the attenuation and
through the intervening precipitation medium. Because of this difference
The transmitted pulse propagates to the resolution volume and back

A.2.1 Propagation Effect

stable except in the most severe cases of propagation contamination.

assumed bias error compensation, ZDR estimation with 0.1 dB accuracy is pos-
sate for these errors. It is shown that with the implementation of the sug-
surement. We consider each factor separately and suggest methods to compen-
s affect the variance of ZDR but introduce a bias error in the ZDR and ZH mea-
which must be investigated for the order of error these factors introduce in
polarization discrimination in the antenna coupler and receiver mismatch.

hardware. There are three important aspects, namely the propagation effect,
while simultaneous sampling makes it possible to obtain highly correlated

In this section...
In terms of the eigenvalues and eigenvectors of the matrix $M$ and can be

The solution of the simultaneous differential equation (4.4) is obtained

angle distribution as well as drop size distribution.

triplation about mean $g$. In general, the elements of $[T]$ depend on the canting

sum canting angle $\alpha$, and are negligible if canting angle has symmetric dis-

the off-diagonal terms $f_{12}$ and $f_{21}$ are zero if all the scatterers have the

the matrix $[T]$ is the forward scatter matrix for this elemental path length.

is the canting angle of the scatterers in the elemental path length $\ell$ and

$$
W_{1} = \begin{bmatrix}
-\sin \phi & \cos \phi \\
\cos \phi & \sin \phi
\end{bmatrix}
$$

(a.6)

Where $K_0$ is the propagation constant for the free space. The matrix $M_1$ is a

$$
\begin{bmatrix}
0 & -jK \\
-jK & 0
\end{bmatrix}
$$

(a.5)

Where $M_0$ is the free space propagation matrix given by $M_0 = -jK M_1$. The 2 x 2 matrix $M$ is the sum of two matrices $M_0$ and $M_1$.

$$
M = \begin{bmatrix}
-\frac{z \rho}{\mu} & 0 \\
0 & -\frac{z \rho}{\mu}
\end{bmatrix}
$$

(4.4)

In order to understand the propagation effect on $Z_2$ XX measurements we

of electric field $E$ through rain is governed by the differential equation

consider the theory of propagation through rain medium breitly. Propagation
For simplicity, we assume that the canting angle of the drops in
change in propagation direction. Since we are evaluating the effect of pro-
transmission matrices whose off-diagonal terms have opposite signs due to the
the reverse propagation from resolution volume to the radar antenna have
a radar the forward propagation from the radar to the resolution volume and
an. The analysis presented so far is for one-way propagation. In the case of
the canting angle and the differential propagation phase shift.

\text{mean canting angle differs from zero. Cross coupling is a function of mean}
\text{the horizontally and vertically polarized transmitted fields, } E^H \text{ and } E^V,
\text{and mean canting angle constant along the entire propagation path. Given}
\text{reflected field components } E^H \text{ and } E^V. \text{With cantain cross-coupled terms if the}
\text{received field components } E^H \text{ and } E^V.

To evaluate the effect of propagation on reflectivity and differential

\text{(1973).}

Rain field substcutions and cascading the solution as proposed by Mccormick
solution can be obtained by dividing the propagation path into several uniform
\text{solution makes the solution of equation (4.4) very difficult. However, an approximate}
\text{trichition, mean canting angle, etc., vary along the propagation path which}
\text{z. In general, the rainfall field is not uniform - the rainfall - drop size dist-}
\text{the solution (4.7) is for uniform rainfall field of propagation path length}
\text{mean canting angle } \theta \text{ and differential phase shift } 

\text{where } [T] \text{ is known as the transmission matrix. The subscript } t \text{ and } r \text{ refer -}

\text{expressed as (9827, 1983).}
At S-band frequencies (3 GHz) the path attenuation is very small (Fang et al. 1987), so we consider only the differential phase shift. The total differential phase shift,

\[
\frac{\langle \chi^Y_{2} \rangle}{\langle \chi^H_{2} \rangle} = \chi^Y_{2} = \chi^H_{2} + \chi^Y_{2}
\]

and

\[
\frac{\langle Z_{2} \rangle}{\langle S_{11} \rangle} = \chi^Z_{2} = \chi^Z_{2} + \chi^Y_{2}
\]

Putting \( E_\ell^Y = E_\ell^H \) into (A.8), we can evaluate the measured reflectivity \( \chi^H \) and measured differential reflectivity \( \chi^Y \) using square law estimators:

and true \( Z_{2} \) and \( Z_{2} \) can be expressed as

\[ E_\ell^H \text{ and } E_\ell^Y \text{ are equal.} \]

For simultaneous \( E_\ell^H \) and \( E_\ell^Y \) sampling scheme (Figure A.7) the transmitted fields

\[ E_\ell^Y = [ E_\ell^Y ] \]

\[ E_\ell^H = [ E_\ell^H ] \]

\[ S_{11} = [ S_{11} ] \]

\[ Z_{2} = [ Z_{2} ] \]

\[ E_\ell^Y \] expressed as

the resolution volume has only diagonal terms. The received fields can then be

the resolution volume is zero and hence the scatter matrix [S] for the volume

(4.9)

(4.10)
expressed as

\[ \text{(A.14) } \theta = \log \left( \frac{H_Z}{H_X} \right) \]

as the gross error in the estimated values of \( H_Z \) and \( H_X \) can be

be assumed to be constant over the sampling interval. From equations (A.9),

The transmission matrix elements are path-integrated values and therefore can

\[ (T_{12} + T_{12}) = 0 \]

\[ (T_{12} + T_{12}) = C \]

\[ (T_{12} + T_{12}) = B \]

\[ (T_{12} + T_{12}) = A \]

where

\[ \text{(A.13) } \frac{Z_{DR}^X}{Z_{DR}^X + Z_{DR}^X} = \frac{\lambda_X}{\lambda_X} = \frac{1}{R_X} \]

and

\[ \text{(A.12) } <S_{II}S_{II} + c*d> + <S_{II}S_{II} + c*d> + <S_{II}S_{II} + c*d> = <S_{II}S_{II} + c*d> = \frac{1}{R_X} \]

\[ \text{(II) } <S_{II}S_{II} + c*d> + <S_{II}S_{II} + c*d> + <S_{II}S_{II} + c*d> = <S_{II}S_{II} + c*d> = \frac{1}{R_X} \]

In terms of the elements of the transmission matrix and the true values of

from (A.9) and (A.10) the estimated values of \( X_H \) and \( X_R \) can be expressed

\[ \theta^* \]

\[ \text{as differential phase shift is a function of path length } z \text{ and rain rate } \theta^* \]
This is equivalent to switching canting angles between +\( \theta \) and \(-\theta\), as far as 45° to the horizontal by changing the phase of the vertical input by 180° and polarization of the transmitted pulse is switched alternately between +45° and -45° measured from horizontal (with a change in sign). An error comes from the bias error with \( \phi \) similar for mean canting angles measurement.

It was observed in the course of bias error computation that the function desirable to compensate for this bias error to increase the usefulness of ZDR at \( \phi = 360°\). Though these kinds of phase shifts are uncommon in nature, it is For beyond 180° the error increases rapidly and can be more than 50% increases from 0° to 90° and then the error decreases to zero when \( \phi = 180°\). Error due to two-way differential propagation phase shift increases as the small at 3 GHz and it would take a propagation path length of at least 40 km, effect (Ough, 1983). Further, differential phase shift per kilometer is to reduce the differential propagation constant thus reducing the propagation is distributed about zero mean. The effect of canting angle distribution for a mean canting angle \( \theta = 0°\). In general, a canting angle in a rain medium can be as much as 50% phase shift of 90° and true ZDR = 3 dB the error in \( \chi = Z\) can be significant for large propagation phase shifts. For a propagation equation (A.13) shows that the bias error in the estimated ZH and ZDR values

\[
\frac{\Delta ZDR}{ZDR} = \frac{1}{10} \log_{10} \left( \frac{ZDR}{ZDR - 100} \right) \quad \text{and} \quad \frac{ZDR}{ZDR} = 10 \log \left( \frac{ZDR - \Delta ZDR}{ZDR} \right)
\]
assumed in the computation.

Various rain rates, uniform rain rate over the entire path length has been

A.18 presents the differential phase shift versus propagation path length for

exponential drop size distribution (N) = (0.88 − 6.0 \times 10^{-3} \cdot m^{-1}). Figure 3 GHz transmission frequency. The values in Figure A.17 are computed using an

tall propagation phase shift per kilometer path length versus rain rate, at

natural we present the theoretically computed values of the two-way differen-

to get an idea of the order of differential phase shifts encountered in

better than that for alternate sampling (Figure 3.7).

pensation error in the is 0.02 dB for θ = 360° and θ = 10°, which is

that the error cancellation is more effective in the than in ZDR. After com-

polarization switch scheme are given in Figures A.15 and A.16. These show

the corresponding bias error in the for 45° polarization and alternate

well as negative.

Figures A.10 through A.14 give the error for different ZDR values, positive as

dramatically reduced from that of +45° transmission alone (Figure A.9).

employed, is a monotonically increasing function of θ, but the error is

of curves. The bias error, when alternate switching of polarization planes is

cancellation is not perfect because of a small asymmetry between the two sets

transmission are nearly symmetric about the zero line. However, the error

decibels. It can be seen that the two sets of curves for +θ and −θ

the true ZDR of the resolution volume is +3dB and the error is expressed in

presented in Figure A.8 as a function of two-way differential phase shift.

The bias error FDR for +45° transmissions for various canting angles is

and -45° polarized transmissions.

expressions (A.10) except that the successive samples are obtained from +45°

the bias error in ZDR and ZH is considered. θR and θH are estimated using
(A.17a)

\[
\frac{\langle e^I \| e \rangle}{\langle e^H \| e \rangle} = \frac{Z_{DR}}{Z_x}
\]

To evaluate this error we set \( Z_{DR} \) to the true \( Z_x \) estimate. Coupled power introduces a bias error in \( Z_{DR} \) estimate.

Power limit to the achievable cross coupling factor and the residual cross.

There is a lower limit to the power flow in the receiver mode. The elements \( A_{11} = A_{22} = 0 \), \( A_{12} = A_{21} = 1.0 \), and \( A_{12} = A_{21} = 0 \) are the inputs and \( A_{11}, A_{22} \) are the outputs (correlated only part of the matrix \( A_{12}, A_{21} \)). The four-port network, we can complete the problem, the matrix is \( 4 \times 4 \). Although the terms of \( e^H, e^V \) are the scatter matrix of the feed and coupler.

(4.16)

\[
\begin{bmatrix}
A_e & Z_{II} \\
A_H & Z_{II}
\end{bmatrix}
\begin{bmatrix}
Z_{II} & A_e \\
Z_{II} & A_H
\end{bmatrix} = \begin{bmatrix}
A^V \\
A^H
\end{bmatrix}
\]

The dual polarization discrimination in the antenna coupler.

A.2.2 Polarization Discrimination in the Antenna Coupler
and using (4.174) we have,

\[
\left(4.19\right)
\]

\[
\frac{1}{\phi} - \frac{1}{\psi} - \frac{1}{\theta} = \frac{1}{\phi}
\]

value \(\phi\). Taking the inverse of matrix \(\left[\alpha\right]\),

Taking the inverse of the coupled scatter matrix \(\left[\alpha\right]\) and using this
detennines \(\left[\alpha\right]\). We can express the true \(\Omega_f\) in terms of the measured
scatter matrix of the coupled can be exactly determined by measurements.

\(\xi\) and hence can be neglected. The phase of \(p\) is an unknown parameter.

Reflection phase shift at the scatterer (this is very small - less than \(0.2\)).

because of the differential propagation phase shift and the differential
section that the correlation coefficient \(\rho^2\) is better than 0.995. However,

For simultaneous sampling of \(E_h\) and \(E_v\), it has been shown in the previous

\(E_h\) and \(E_v\) samples.

where \(\Re\{\cdot\}\) indicates real part and \(\phi\) is the correlation coefficient between

\[
\left(4.18\right)
\]

\[
\frac{1}{\phi} + \frac{1}{\psi} + \frac{1}{\theta} = \frac{1}{\phi}
\]

Using (4.15), \(\phi\) and \(\psi\) can be related via equation,

\[
\left(4.16\right)
\]

\[
\frac{<\bar{Z}\psi|\bar{Z}\psi>}{<\bar{Z}\psi|\bar{Z}\psi>} = \frac{1}{\phi}
\]
Received signals, $E_H$ and $E_V$ are switched between the receivers $R_1$ and $R_2$. The schematic of the hardware is shown in Figures A.19 and A.20. The error in the estimated $Z_{DR}$ due to receiver mismatch is shown that with this scheme bias error less than 0.1% plas a 10% mismatch in the gains of two receivers translates to less than 0.1% bias error in $Z_{DR}$. Suggested here is a scheme which reduces the mismatch between $E_H$ and $E_V$ samples by being measured. Because meteorological signals have large dynamic range ($>80$ dB), perfect phase and gain matching of the receivers over the entire dynamic range is very difficult. Power gain has to be matched to within 0.1 dB (approximately 2.5% accuracy to maintain $Z_{DR}$) for accurate measurement of $Z_{DR}$ the two receivers (see Figure A.7) have ortho-mode coupler for less than -35 db cross coupling. It should be possible in practice to fine tune the made extremely stable, it should be possible to fine tune the frequency and the frequency can be applied, because radar operaters at a fixed frequency and $Z_{DR}$ estimate can be made using $A_{ZDR}$ to an added factor, and further improvement in $Z_{DR}$ estimate can be made using $A_{ZDR}$. It is desirable to reduce to a lowest possible value by fine tuning the estimate along with $X_D$. If $A$ is determined accurately, the error in $Z_{DR}$ can be corrected using $A_{ZDR}$.

In $A_{ZDR}$ the correlation coefficient $A$ is between $X_D$ and $Y_D$ which can be

$$A_{ZDR} = \frac{X_D (Z_{DR} \text{real})}{X_D (Z_{DR} \text{real}) + Y_D (Z_{DR} \text{real})}$$
\[ A_{d} \frac{W}{Z} + V_{d} \frac{W}{g} = < A^V A^V > = A_{Z} \]

and

\[ H_{d} \frac{W}{Z} + H_{d} \frac{W}{g} = < H^V H^V > = H_{Z} \]

The estimated horizontal and vertical reflectivities are proportional to

\[ V^V \quad E = 1 \quad Z = 2, 4, 6 \]

\[ V^V \quad E = 5 \quad Z = 1, 3, 5 \]

If \( E^H \) and \( E^V \) are the horizontal and vertical signal inputs, the correlation between the power gain of receiver \( R \) be \( G_1 = G_2 = G + 6 \), so that the subsequent circuits need not be altered.

With \( S_1 \), switches the output to the proper horizontal and vertical channels.

Alternately with each pulse transmission, a second switch \( S_2 \) in synchronism
Fractional errors $a$ and $b$ are equal. If the correlation coefficient is less than $1.0$, the measured value $\chi_{DR}$ remains accurate when $|a| \ll |b|$, which tend to zero for large values of $a$ and $b$ are fractional errors. Where $a$ and $b$ are fractional errors, $|a| < |b|$, which tend to zero for large $a$, and $b$ are fractional errors.

\[
\frac{VI_d}{Z_d} = \frac{VI_d}{2Z_d^2} (1 + b)
\]

and

\[
\frac{\chi_{DR}}{Z_d} = \frac{VI_d}{2H_d^2} (1 + a)
\]

For some finite $M$, we can write:

**Zero bias error.**

... of $\chi_{DR}$ tends to zero and unity respectively, making $\chi_{DR} = \chi_{DR}^0$. For sufficiently large $M$, the ratio of summations in the numerator and denominator:

\[
\frac{\sum_{z} |A_{z}|^2}{\sum_{z} |H_{z}|^2} = \frac{\sum_{z} |A_{z}|^2}{\sum_{z} |H_{z}|^2} = \chi_{DR}
\]

Denoting the estimated value of differential reflectivity as $\chi_{DR}$,

\[
\frac{VI_d}{Z_d} = \chi_{DR}
\]

The true value of the differential reflectivity at the input of the receiver.
pair processing is used.

Bias error in the mean velocity due to phase mismatch in receivers, when pulse switching once in two pulse transmissions. This procedure also removes the rate has to be half that of polarization switching. The receivers are not used to avoid the measured $\Delta\theta_{P}$.

In this report (Figure A.27) the success of simultaneous measurement of $E_{H}$ and $E_{V}$ is ideal. In the $E_{H}$ and $E_{V}$ sequence as obtained in this analysis of receiver switching we have assumed that the input approximately 0.1%.

For a 10% error in receiver gain match (i.e., $g_{0} = 0.1$) and 10% difference in $E_{H}$ and $E_{V}$, the error in measured differential reflectivity $\Delta R_{\theta}$ is

$$\Delta R_{\theta} = \frac{I_{+} + I_{-}}{2} \cdot \frac{g_{0}}{g_{0} + 1} \cdot \frac{1}{\theta_{P}}$$

(A.26)

To some residual error $\Delta R_{\theta}$ given by

Then unity, the fractional errors are different in general. This gives rise
Fig. A.1 Standard deviation, $\sigma_{DR}$, versus number of pairs, $M$, for simultaneous sampling of $E_H$.

Fig. A.2 Standard deviation, $\sigma_{DR}$, versus $M$ for simultaneous sampling of $E_V$.

Fig. A.3 Standard deviation, $\sigma_{DR}$, versus $E_V$, $|p_{HV}(0)|^2 = 0.99$.

Fig. A.4 Axis ratio $a/b$ versus $E_V$, $|p_{HV}(0)|^2 = 0.99$.

Fig. A.5 Truncated Gaussian distribution of $a/b$ ratio.

Fig. A.6 Pair correlation coefficient variation with rainfall rate $R$ for oscillation model; standard deviation $\sigma_0$ of canting angle distribution, variation of $|\rho(0)|^2$ with spectrum width.

Fig. A.7 Block diagram for two-way differential propagation.

Fig. A.8 Bias error, $\Delta Z_{DR}$, for transmission at $45^\circ$ polarization and simultaneous sampling of $E_H$ and $E_V$. Mean canting angle $\theta$ is shown as a parameter. True value of $\theta$ is $3^\circ$.
Fig. A.19. Scramatric of receiver switching for mismatch compensation.

Fig. A.20. Parameter.

Fig. A.21. Two way differential phase shift versus path length z with k as a parameter.

Fig. A.22. Two way differential phase shift per km versus rational rate k.

Fig. A.23. Transmission; ZDR = 3 dB.

Fig. A.24. Bias error GzH versus after compensation; ZDR = 3 dB.

Fig. A.25. Parameter for 45° polarized transmission; ZDR = 3 dB.

Fig. A.26. Bias error GzH versus two way propagation phase shift with g as a parameter.

Fig. A.27. GzDR versus after compensation; ZDR = -3 dB.

Fig. A.28. GzDR versus after compensation; ZDR = -2 dB.

Fig. A.29. GzDR versus after compensation; ZDR = -1 dB.

Fig. A.30. GzDR versus after compensation; ZDR = 1 dB.

Fig. A.31. GzDR versus after compensation; ZDR = 2 dB.

Fig. A.32. Between 45° polarizations alternatively; ZDR = 3 dB.
Fig. 2.1 The geometry of raindrop scattering.
Fig. 2.2 Relation between differential reflectivity, $Z_{DR}$, and rainfall rate.

For exponential DSD, the Marshall-Palmer distribution corresponds to $N_0 = 8000 \text{ m}^{-3} \text{mm}^{-1}$.
\text{FIG. 3.1 Standard deviation of } z \text{, } \sigma_R, \text{ versus numbers of sample pairs, } M. \text{ For } |\rho(0)|^2 = 1.0, $\text{VAR}(z) = \frac{\sigma_R^2}{\sigma^2}$. \text{Unambiguous Vel. } v_d = 25 \text{ m/s}$.
Fig. 3.2 Standard deviation of $\tilde{Z}_{DR}$, $q_{DR}$, versus numbers of sample pairs, $M$, for $|\rho_{HV}(0)|^2 = 0.995$. 

Alternate sampling corr. coeff. $|\rho_{HV}(0)|^2 = 0.995$

Unambiguous vel. $v_0 = 25$ m s$^{-1}$

Spectrum width $\sigma_v = 6$ m s$^{-1}$
FIG. 3.3 Standard deviation of ZDR, dBZ, versus numbers of sample pairs, M.

For $|\rho_{\text{HH}}(0)|^2 = 0.99$. 

Number of sample pairs, M

Standard deviation, $\sigma_{\text{DR}}$, (dB)
Fig. 3.4 Standard deviation of $\tilde{v}_R$, $\tilde{v}_R$ versus numbers of sample pairs, $M$. for $|\hat{p}_{\omega}(0)|^2 = 0.98$. 

Alternate sampling corr. coeff. $|\hat{p}_{\omega}(0)|^2 = 0.98$

Unambiguous vel. $v_0 = 25 \text{ m s}^{-1}$

Spectrum width $v = 1, 2, 3, 4, 5, 6$
FIG. 3.5 Bias error in horizontal reflectivity due to propagation with alternate sampling scheme.

Two-Way Differential Phase Shift (deg)

Bias Error, $S_2H$ (dB)

Canting Angle $\theta = 0^\circ$

$Z_{0R} = 3$ dB

Alternate Sampling
With alternate sampling scheme, Fig. 3.6 bias error in differential reflectivity, $\angle DR$, due to propagation.
Fig. 3.7 Alternate transmission sequence. Correlations of interest are indicated.
Fig. 38. Possible transmission sequences for NEXRAD.
By alternate sampling of $E_h$ and $E_v$ signals.

Figure 3.9 Block diagram of the dual linear polarized radar for ZDR measurement.

Cost Estimates
Fig. 3.10 Block diagram of the circular polarized radar for CDR measurement.

- Signal processor
- Receiver
- Circular/Orthomode transducer
- Scalar feed
- Cost estimates
- Transmitter
- Circularator
- Antenna

$20K$
$5K$
$5K$
$20K$
$20K$
Sampling of \( C_H \) and \( C_V \).
Figure 3.12 Standard deviations of differential reflectivity. DR with simultaneous and alternate sampling.
Sampling of $E_h$ and $E_v$. $|V_H^\prime(0)|^2 = 0.995$.

Fig. A.1 Standard deviation, $\sigma_{DR}$, versus number of sample pairs, $N$, for simultaneous

Unambiguous Vel. $\psi = 25\ m/s$.
Corr. Coef. $|V_H^\prime(0)|^2 = 0.995$
Simultaneous Sampling
and \( \frac{\phi}{\sqrt{V}} = 0.99 \).

**Figure A.2.** Standard deviation, \( \sigma_{DR} \), versus \( M \) for simultaneous sampling of \( E_{H} \).

The graph shows the relationship between the number of sample pairs, \( M \), and the standard deviation, \( \sigma_{DR} \), with spectrum width and unambiguous velocity, \( v_o = 25 \text{ m/s}^{-1} \), and correlation coefficient, \( \rho_{HY} = 0.99 \).
Fig. A.3 Standard deviation, $\sigma_{DR}$, versus $M$ for simultaneous sampling of $E_H$ and $E_V$, $|\rho_{HV}(0)|^2 = 0.98$. 

SIMULTANEOUS SAMPLING, CORR. COEFF. $|\rho_{HV}(0)|^2 = 0.98$

SPECTRUM WIDTH

UNAMBIGUOUS VEL. $v_0 = 25\,\text{m s}^{-1}$

NUMBER OF SAMPLE PAIRS, $M$

STANDARD DEVIATION, $\sigma_{DR}$ (dB)
Fig. A.4 Axis ratio a/b versus equilvolumetric drop diameter D.

(c) Upper and lower limits of a/b for oscillating drops.

(a) Equilibrium shape, (b) time averaged a/b for oscillating drops.
Fig. A.5 Truncated Gaussian distribution of $a/b$ ratios.
Distribution: variation of \(|p(1)|^2\) with spectrum width.

Oscillation model: standard deviation \(\theta\) of canting angle.

Fig. A.6 Pair correlation coefficient variation with rainfall rate \(R\) for

\[\text{DOPPLER SPECTRUM WIDTH } \theta \text{ : m/s}^{-1}(T_s = 780 \mu s)\]

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>Width of Canting Angle Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90°</td>
</tr>
<tr>
<td>4.5°</td>
<td>13.5°</td>
</tr>
<tr>
<td>9.0°</td>
<td>18.0°</td>
</tr>
<tr>
<td>13.5°</td>
<td>22.5°</td>
</tr>
<tr>
<td>18.0°</td>
<td>27.0°</td>
</tr>
<tr>
<td>22.5°</td>
<td>31.5°</td>
</tr>
<tr>
<td>27.0°</td>
<td>36.0°</td>
</tr>
</tbody>
</table>

Rain Rate, \(R\) (mm/hr)

Correlation Coefficient \(|p_{hw}|^2\)
Fig. A.7 Block diagram of radar for simultaneous sampling of $E_H$ and $E_V$. 

[Diagram showing a block diagram of a radar system with labels for transmitter, receiver 1, receiver 2, ADC, power divider, phase shifter, orthomode coupler, and antenna.]
Parameter: True value of Z DR is 3 dB.

Sampling of E and E\textsuperscript{+} Mean canting angle \( \theta \) is shown as a phase shift \( \phi \) for transmission at +45° polarization and simultaneous

Fig. A-8. Bias error ZDR variation with two-way differential propagation.

\[ \text{BIAS ERROR, ZDR (dB)} \]

\[ \text{TWO-WAY DIFFERENTIAL PHASE SHIFT (deg)} \]

\[ \theta = 8\text{°} \]

\[ \text{CANTING ANGLE} \]

\[ \text{POLARIZATION} +45\text{°} \]

\[ \text{SIMULTANEOUS} \]
Fig. A.9 Bias error $\delta_{DR}$ after compensation. Transmission is switched between $\pm45^\circ$ polarizations alternately; $Z_{DR} = 308$. 

$Z_0 = 3$ dB

ALTERNATE $\pm45^\circ$
Polarizations

CANTING ANGLE, $\delta = 10^\circ$

TWO-WAY DIFFERENTIAL PHASE SHIFT, $\phi$ (radian)
Fig. A.10 60° vs. 87° after compensation: ZDR = 2 dB.

TWO-WAY DIFFERENTIATED PHASE SHIFT (deg)

ALTERNATE 145° POLARIZATION
ANGLE $\theta = 10^\circ$
CANTING

BIAS ERROR, 87 (dB)
FIG. A.11  ZDR Versus ϕ after compensation; ZDR = 1 dB.
Fig. A.12 2DR versus φ after compensation; 2DR = -1 dB.

Two-Way Differential Phase Shift (deg)

Bias Error, 2DR (dB)

Canting Angle: φ = 10°

Alternate ±45° Polarization

2DR = -1 dB
Fig. A.13  $\Delta Z$DR versus after compensation; $Z_{DR} = -2$ dB.

Two-Way Differential Phase Shift, $\phi$ (deg)

Bias Error, $Z_{DR}$ (dB)

Alternate $\pm 45^\circ$ Polarization

Canting Angle, $\theta = 10^\circ$
Fig. A.14  EZDR versus θ after compensation; ZDR = -3 dB.
A parameter for +45° polarized transmission $Z_{0R} = 3$ dB.

Fig. A.15 Bias error $\Delta Z \overline{H}$ versus two way propagation phase shift with $\theta$ as

TWO-WAY DIFFERENTIAL PHASE SHIFT (deg)

BIAS ERROR, $\Delta Z \overline{H}$ (dB)

+45° POLARIZATION

SIMULTANEOUS SAMPLING

CANTING ANGLE

$\theta = \theta^\circ$
Fig. A.16 Bias error $\theta_{zH}$ versus phase compensation at 45° polarized transmission; ZDR = 3 dB.

TWO-WAY DIFFERENTIAL PHASE SHIFT, $\phi$ (deg)

BIAS ERROR, $8z_{H}$ (dB)

CROSSTALK ANGLE, $\theta = 10$°

ALTERNATE 45° POLARIZATION

ZDR = 3 dB
FIG. A.II. Two-way differential phase shift per km versus rainfall rate R.
Figure A.18: Two-way differential phase shift $\phi$ versus path length $z$ with $R$ as a parameter.
Fig. A.19 Schematic of receiver switching for mismatch compensation.
Fig. A.20 Block diagram of a radar with receiver switching.